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Energy-Efficient Routing in Linear Wireless Sensor Networks

Seminar Paper
(Großer Beleg)

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Abstract

Wireless sensor networks are used for structure monitoring and border surveillance. Typical applications, such as sensors embedded in the outer surface of a pipeline or mounted along the supporting structure of a bridge, feature a linear sensor arrangement. Economical power use of sensor nodes is essential for long-lasting operation.

In this paper, we present MERR (Minimum Energy Relay Routing), a novel approach to energy-efficient data routing to a single control center in a linear sensor topology. Based on an optimal transmission distance, relay paths are established that aim for minimizing the total power consumption. We study MERR by both stochastic analysis and simulation, comparing it to other possible approaches and a theoretically optimal protocol. We find that MERR consumes 80% less power than conventional approaches and performs close to the theoretical optimum for practicable sensor networks.

Kurzfassung

Drahtlose Sensornetzwerke werden zur Kontrolle und Überwachung von Bauwerken und Grenzzonen eingesetzt. So findet man zum Beispiel Fernleitungen, in deren Außenwand Sensoren eingebaut sind oder Sensoren entlang der tragenden Elemente von Brücken. Diese Anwendungsfälle zeichnen sich im Besonderen durch eine lineare Sensortopologie aus. Unabhängig davon ist grundsätzlich eine sparsame Energienutzung notwendig um eine lange Betriebsdauer des Netzwerkes zu gewährleisten.

In der vorliegenden Arbeit stellen wir MERR (Minimum Energy Relay Routing) vor. Dies ist ein neuartiger Ansatz, welcher energieeffizientes Routing von Sensordaten zu einer einzelnen Kontrollstation in einer linearen Sensortopologie ermöglicht. Basierend auf einer optimalen Übertragungsdistanz die Pfade so ermittelt, dass der Gesamtenergieverbrauch minimiert wird. Wir untersuchen MERR mittels stochastischer Methoden und Simulation. Dabei vergleichen wir es mit anderen möglichen Vorgehensweisen sowie einem theoretisch optimalen Protokoll. Wir stellen fest, dass MERR 80% weniger Energie als konventionelle Protokolle verbraucht und für praktisch realisierbare Sensornetzwerke dem theoretischen Optimum nahe kommt.

Contents

Chapter 1

Introduction

Wireless sensor networks are a collection of small sensing devices that collaborate to gather information of some physical phenomenon. These battery-powered devices can be equipped with many different sensors, are capable of local data processing, and communicate with each other over a wireless channel. In many applications, a good portion of the obtained information is collected at a control center, usually called a *base station*. Sensor nodes preprocess and exchange information ultimately destined for the base station whose responsibility is to analyze the data and take appropriate action.

Wireless networks of sensors are used for environmental monitoring, health, military, home, and other commercial applications such as inventory control [ASSC02]. In the home of the future, aeration and heating might be controlled based on readings of temperature and humidity sensors. In health, sensor nodes can also be deployed to assist disabled patients or to monitor medical equipment in order to locate it quickly in an emergency.

The realization of these and other sensor network applications require protocols that account for the limited energy, computational power, and communication resources available to the sensors. A protocol should be energy-aware because in case of a dead sensor node, no information can be obtained from this particular area anymore. Replenishment of batteries is costly or even impossible. The limited computational power of sensors demands for simple network protocols, and limited bandwidth of the wireless links puts constraints on the communication between sensor nodes.

Many routing protocols have been designed for wireless sensor networks [AKK04]. The vision behind these protocols is usually a network of up to thousands of nodes randomly deployed throughout a sensor field that operate in a mesh topology. For specific application scenarios, however, a mesh topology may not be appropriate or simply not feasible. This can be due to physical structure, measuring point distribution or other criteria. Consider, for example, a bridge where sensors are mounted at each pillar to monitor the supporting structure, sensors deployed along the outskirts of a forest to observe the movement of wild game, or encroachment control of pipelines with sensors embedded in the outer surface. Here, the positions of sensor nodes and hence their linear topology is predetermined by the present physical structure and application requirements.

In this paper, we present MERR (Minimum Energy Relay Routing), a novel routing protocol for linear wireless sensor networks. Assuming homogeneous sensor nodes, MERR enables energy-efficient delivery of sensor data to a single base station.

There are two conventional approaches to establish a link between a sensor and the base station in a linear network: direct transmission and MTE (minimum-transmission-energy) routing. With the former model, a sensor transmits directly to the base station, while the latter approach uses all nodes between a sensor and the base station for multi-hop routing. From an energy perspective, neither of them is perfect. Direct transmission drains significant power of nodes far away since the radio signal attenuates as a function of transmission distance raised to some power [Rap96]. In an environment with many obstacles or for very long distances, direct transmission may not be feasible at all. MTE routing consumes immoderate receive energy when nodes are close to each other.

In MERR, sensor data is routed to the base station using intermediate relay nodes. The relays are selected such that the distances between them are approximately equal to a *characteristic distance* [BGC01]. This distance is a constant and can be thought of an optimal transmission range where the total power needed for routing is minimized. Unlike MTE routing, not all intermediate nodes are used as relays. In fact, some nodes may be left out between successive relays in order to get as close as possible to the characteristic distance.

We evaluate our proposed protocol by both theoretical analysis and simulation using a stochastic model for the distribution of sensors on a line. As our results show, MERR achieves power savings of 80% compared to MTE routing if the mean distance between adjacent sensors is one eighth of the characteristic distance[1]. We also find that MERR deviates less than 10% from the theoretical optimum if the mean distance is smaller than half and one third of the characteristic distance for path loss exponent 2 and 4, respectively.

The thesis at hand comprises three main parts. The first part, laying the foundations for our proposed protocol, comprises Chapters 2 and 3. We describe

[1] For our radio parameters, the characteristic distance is equal to 100 m and 71 m for path loss exponent 2 and 4, respectively.

the models we build upon, identify the objective of this work and review existing routing approaches. Moreover, the problem of optimal routing in linear sensor networks is discussed in detail. The second part, introducing the MERR protocol, comprises Chapters 4 and 5. We present concept and implications of MERR followed by the evaluation of its performance compared to conventional approaches and a theoretically optimal protocol. Chapters 6 and 7, which comprise the third part, put this papers contribution into perspective of previous work and make some concluding statements.

Chapter 2

System Model and Problem Statement

We present the type of network discussed in this paper and formulate the problem we aim to solve.

2.1 Node Model

In this work, we consider a network (see Figure 2.1) composed of n sensor nodes distributed on a line and one base station (BS) located at the left-most position. Each sensor node has a unique ID starting with node 1 right of the base station.

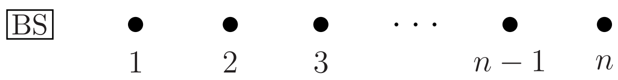


Figure 2.1: A linear network of n sensors and one base station.

A node can be classified as *live* or *dead* depending on whether it has any energy left or not. In this model, introduced in [BGC01], a live node plays one of three roles that can change dynamically with time:

- Sensor: The node senses the environment and produces data that needs to be relayed to the base station.
- Relay: The node forwards received data onward without any processing.
- Powered down: The node does not participate in either sensing or relaying.

The ultimate destination of any data produced by a sensor is the base station. No data aggregation or caching is performed by a relay.

We assume static sensor nodes that are aware of the distances to downstream[1] nodes within their maximum transmission range. In case of manual sensor deployment, these distances can be deduced from the respective sensor positions and then advertised in the network during a setup phase. Another approach is distance estimation by nodes using received signal strength (RSS), time of arrival (ToA), or similar methods [WWT].

Furthermore, we make the assumption that all sensor modules have equal power, computational, and communicational resources, and that the hardware is capable of adjusting the transmission power dynamically. This is supported, for example, by Chipcon's CC2420 transceiver whose output power is programmable in 8 steps from approximately −24 dBm to 0 dBm [chi07]. Sensors adjust their output power in order to achieve a certain transmission range. In this way, they can also control the power consumption of their transceiver unit.

2.2 Radio Model

For our theoretical analysis and experiments we adopt the radio model as it is used in [BGC01, HCB00, Hei00]. The key energy parameters are the energy needed to receive a bit (E_{rx}) and transmit a bit over a distance d (E_{tx}). Assuming that the received power decays as a function of the distance d between transmitter and receiver raised to the power of γ [Rap96], we have

$$E_{rx} = \alpha_{rx} \tag{2.1}$$

$$E_{tx} = \alpha_{tx} + \epsilon\, d^{\gamma}. \tag{2.2}$$

Here, α_{rx} and α_{tx} are the energy/bit consumed by the receiver and transmitter electronics, respectively, and ϵ accounts for the energy dissipated in the transmit amplifier. Hence, the power needed by a relay when receiving data and then transmitting it a distance d onward is given by

$$\begin{aligned} P_{relay}(d) &= (\alpha_{rx} + \alpha_{tx} + \epsilon\, d^{\gamma})\, r \\ &\equiv (\alpha + \epsilon\, d^{\gamma})\, r, \end{aligned} \tag{2.3}$$

where r is the number of bits relayed per second. Since P_{relay} scales linearly with r, we omit this term in the following and implicitly assume $r = 1$ bit/s.

[1] *Downstream* is a synonym for "in the direction of the base station." *Upstream* refers to the opposite direction.

2.3 Linear Routing Problem

Because the sensor modules are powered by finite energy sources that are costly or even impossible to replenish, energy-efficiency is a key design criterion for both sensor hardware and protocols [ASSC02]. In this paper, we tackle the problem of low-power routing in sensor networks with linear topology as it is stated in the following proposition:

Linear Energy-Efficient Routing Problem: *Given a network of n sensors haphazardly distributed on a line where the base station, located at one end of the linear network, collects the data of all sensors, and each sensor is aware of the distances to its downstream neighbors within reach. We claim that the MERR protocol conveys sensor data more energy-efficient than conventional routing schemes and performs close to the theoretical optimum.*

Chapter 3

Conventional Approaches and Optimal Routing

We review direct transmission and MTE routing as two conventional approaches to collect data at a base station in linear sensor networks, followed by a discussion of optimal routing paths. Direct transmission and MTE routing are special cases of MERR and MERR approaches optimal routing as we define it.

3.1 Direct Transmission

The simplest way of communication between nodes in a sensor network and the base station is over a direct link. Using direct transmission, each sensor sends its data directly to the base station; no other nodes are involved in the transmission process. This is shown for a linear sensor network in Figure 3.1.

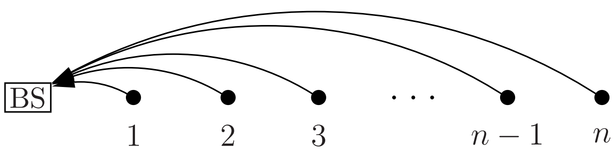


Figure 3.1: Direct transmission. Nodes send their data directly to the base station.

With direct transmission, the batteries of nodes far away from the base station will quickly drain since transmission power increases as a power function of the distance between transmitter and receiver. In an environment with many obstacles or if the distance is too large, successful reception might not be feasible at all. In this case, the received signal strength is below the sensitivity threshold which makes error-free demodulation and decoding impossible. If nodes are close to the base station or the energy required for reception is large, direct transmission can be the method of choice because no receive energy is dissipated. The only receptions occur at the base station which is normally assumed to have unlimited power supply.

3.2 MTE Routing

Another approach to convey data is by the use of other nodes. Intermediate nodes route other sensors' data that is destined for the base station. In MTE routing, these routers are chosen such that the transmit amplifier energy (ϵd^{γ}) is minimized [HCB00]. The energy dissipated in the receiver circuitry (α_{rx}) is disregarded. Thus, node A would transmit to node C through node B if and only if

$$d_{AB}^{\gamma} + d_{BC}^{\gamma} \; < \; d_{AC}^{\gamma}. \tag{3.1}$$

Running MTE on sensors forming a linear network causes each sensor to transmit to its direct downstream neighbor. If, as shown in Figure 3.2, sensor n wants to deliver its data, all $(n-1)$ nodes between sensor n and the base station will be used as relays.

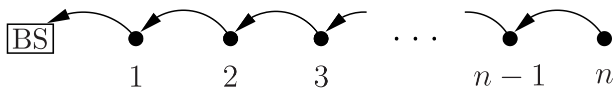


Figure 3.2: MTE routing. All intermediate nodes are used as relays.

Multi-hop routing is preferable for long-distance transmissions. It can dramatically reduce transmission power compared to direct communication. The drawback of MTE routing is that immoderate receive energy is consumed if nodes are close to each other or the energy required for reception is high.

3.3 Optimal Routing

After discussing two conventional approaches, we now turn to the problem of optimal routing in a linear sensor network. We formulate this problem as follows: assume a single sensor, located at distance D from the base station, and we can place as many relays as we want anywhere in between. What is the optimal number of relays and how are they best arranged to minimize the total relaying energy?

Bhardwaj, Garnett, and Chandrakasan [BGC01] show that the optimal number of hops (K_{opt}) is always one of

$$K_{opt} = \left\lfloor \frac{D}{d_{char}} \right\rfloor \text{ or } \left\lceil \frac{D}{d_{char}} \right\rceil \tag{3.2}$$

where d_{char} is the characteristic distance given by

$$d_{char} = \sqrt[\gamma]{\frac{\alpha}{\epsilon\,(\gamma - 1)}}. \tag{3.3}$$

Furthermore, the total relaying energy is minimized when all the hop distances are made equal to D/K_{opt}. This means, we should space $(K_{opt} - 1)$ relays in constant intervals of D/K_{opt} to establish an optimal routing path.

When using (3.2) to determine the optimal number of hops K_{opt}, two possible cases can arise. First, if D is an integral multiple of d_{char}, the result of D over d_{char} will be an integer. In this trivial case, K_{opt} follows directly (see Figure 3.3). Second, if D is not an integral multiple of d_{char}, D over d_{char} will be a floating point number. Then we have two possible values to be considered for K_{opt}, depending on whether we apply the floor or the ceiling function to the result. But which function and thus which number of hops will be the optimal choice, that is, will result in the most energy-efficient relaying process?

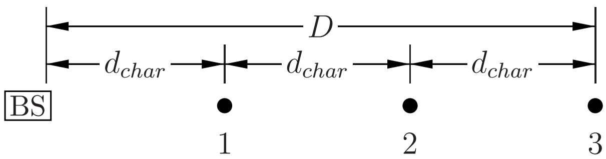


Figure 3.3: Optimal routing. In the trivial case that D is an integral multiple of d_{char}, the optimal number of hops K_{opt} is given by D/d_{char}. Here, $K_{opt} = 3$ and the relays 1 and 2 are spaced in intervals of d_{char} to make the routing path $3 \to 2 \to 1 \to$ BS optimal.

3.3.1 Derivation of Criterion

In order two derive a criterion that tells us which number to choose, we first express the distance D to the base station as a multiple of the characteristic distance d_{char} as follows,

$$D = (m + \delta)\, d_{char}, \tag{3.4}$$

where $m \in \mathbb{N}$, $m \geq 0$, $\delta \in \mathbb{R}$, $0 \leq \delta < 1$. Note that m is the integral part and δ is the fractional part of D/d_{char}. The position of the base station is given when $m = 0$

and $\delta = 0$. The case $m > 0$ and $\delta = 0$ refers to the trivial case discussed above where D is an integral multiple of d_{char} ($D = m\, d_{char}$). The optimal number of hops K_{opt} follows directly.

With (3.4) the two alternatives in (3.2) simplify to

$$K_{floor} = \left\lfloor \frac{D}{d_{char}} \right\rfloor = m + \lfloor \delta \rfloor = m \tag{3.5}$$

$$K_{ceil} = \left\lceil \frac{D}{d_{char}} \right\rceil = m + \lceil \delta \rceil = m + 1. \tag{3.6}$$

where the floor and ceiling function are eliminated. In case we have a node whose distance D to the base station is smaller than d_{char}, that is, $m = 0$ and $0 < \delta < 1$ ($D = \delta\, d_{char}$), the optimal number of hops is either $K_{floor} = 0$ or $K_{ceil} = 1$. However, only $K_{opt} = K_{ceil} = 1$ is feasible because the data must be delivered to the base station. Thus, the data is directly transmitted to the base station. From now on we consider the case where $m > 0$ and $0 < \delta < 1$. We have always two possibilities for K_{opt}, K_{floor} or K_{ceil}, and have to choose one.

We can make this decision based on the comparison of the respective rate of energy dissipation. Let $P_{floor}(D)$ denote the power needed to relay data with unit rate using K_{floor} hops, and $P_{ceil}(D)$ the respective power using K_{ceil} hops. If we compare both power rates, we can state the following criterion:

$$K_{opt} = \begin{cases} K_{floor} & \text{, if } P_{floor}(D) \leq P_{ceil}(D) \\ K_{ceil} & \text{, if } P_{floor}(D) > P_{ceil}(D) \end{cases} \tag{3.7}$$

But how can we compute P_{floor} and P_{ceil}? In [BGC01], the authors derive a lower bound for the power (P_{link}) dissipated in relaying a bit over a distance D using K hops,

$$\begin{aligned} P_{link}(D) &\geq K\, P_{relay}\left(\frac{D}{K}\right) - \alpha_{rx} \\ &= K\left[\alpha + \epsilon \left(\frac{D}{K}\right)^{\gamma}\right] - \alpha_{rx}, \end{aligned} \tag{3.8}$$

with P_{relay} given by (2.3). Equality occurs if and only if all hop distances are equal to D/K. Provided that the radio parameters α and ϵ, the path loss exponent γ, and D are known, we would first compute the characteristic distance d_{char}, then determine K_{floor} and K_{ceil}, and finally use (3.8) to compute P_{floor} and P_{ceil}. Afterwards, the decision whether K_{floor} or K_{ceil} leads to minimum power consumption can be made by comparing P_{floor} with P_{ceil}.

In foresight that a sensor node in a real application scenario might implement (3.7), it would be better not to rely on all the different parameters necessary to compute P_{floor} and P_{ceil}. It is rather desireable to design a distributed protocol that requires as less knowledge as needed to fulfill its intended function. We therefore deduce in the following a criterion that requires only the knowledge of D and d_{char}. For static sensor nodes the distance D to the base station is constant and also the propagation environment, reflected through γ, stays the same. We thereby concentrate on the frequently assumed path loss exponents 2 and 4.

Replacing K in (3.8) by (3.5) and D by (3.4) leads to

$$\begin{aligned} P_{floor} &\geq K_{floor}\, P_{relay}\left(\frac{D}{K_{floor}}\right) - \alpha_{rx} \\ &= m\left[\alpha + \epsilon\, d_{char}^{\gamma}\left(\frac{m+\delta}{m}\right)^{\gamma}\right] - \alpha_{rx} \\ &= m\left[\alpha + \epsilon\,\frac{\alpha}{\epsilon(\gamma-1)}\left(\frac{m+\delta}{m}\right)^{\gamma}\right] - \alpha_{rx} \\ &= \alpha\, m\left[1 + \frac{\left(\frac{m+\delta}{m}\right)^{\gamma}}{\gamma-1}\right] - \alpha_{rx}, \end{aligned} \tag{3.9}$$

with equality if and only if all the hop disances are equal to D/K_{floor}.

Similarly, if we replace K in (3.8) by (3.6) and D by (3.4), we get

$$\begin{aligned} P_{ceil} &\geq K_{ceil}\, P_{relay}\left(\frac{D}{K_{ceil}}\right) - \alpha_{rx} \\ &= (m+1)\left[\alpha + \epsilon\, d_{char}^{\gamma}\left(\frac{m+\delta}{m+1}\right)^{\gamma}\right] - \alpha_{rx} \\ &= (m+1)\left[\alpha + \epsilon\,\frac{\alpha}{\epsilon(\gamma-1)}\left(\frac{m+\delta}{m+1}\right)^{\gamma}\right] - \alpha_{rx} \\ &= \alpha\,(m+1)\left[1 + \frac{\left(\frac{m+\delta}{m+1}\right)^{\gamma}}{\gamma-1}\right] - \alpha_{rx}, \end{aligned} \tag{3.10}$$

with equality if and only if all the hop distances are equal D/K_{ceil}. We now compare these expressions and want to find out, if P_{floor} is greater than P_{ceil}. The domain is

$m \in \mathbb{N}$, $m > 0$, $\delta \in \mathbb{R}$, $0 < \delta < 1$, and $\gamma = \{2, 4\}$.

$$\begin{aligned}
P_{floor} &> P_{ceil} \\
\alpha\, m \left[1 + \frac{\left(\frac{m+\delta}{m}\right)^{\gamma}}{\gamma - 1}\right] - \alpha_{rx} &> \alpha\,(m+1)\left[1 + \frac{\left(\frac{m+\delta}{m+1}\right)^{\gamma}}{\gamma - 1}\right] - \alpha_{rx} \\
m\left(\frac{m+\delta}{m}\right)^{\gamma} &> \gamma - 1 + (m+1)\left(\frac{m+\delta}{m+1}\right)^{\gamma} \\
1 - \gamma &> (m+1)\left(\frac{m+\delta}{m+1}\right)^{\gamma} - m\left(\frac{m+\delta}{m}\right)^{\gamma} \\
1 - \gamma &> (m+1)^{1-\gamma}(m+\delta)^{\gamma} - m^{1-\gamma}(m+\delta)^{\gamma} \\
1 - \gamma &> (m+\delta)^{\gamma}\left[(m+1)^{1-\gamma} - m^{1-\gamma}\right] \\
\frac{1-\gamma}{(m+1)^{1-\gamma} - m^{1-\gamma}} &< (m+\delta)^{\gamma} \qquad (3.11)
\end{aligned}$$

Note that the relation symbol changes in the last step because the term $[(m+1)^{1-\gamma} - m^{1-\gamma}]$ is negative in the domain.

3.3.2 Criterion for Quadratic Attenuation Model

For the case $\gamma = 2$, (3.11) simplifies to

$$\begin{aligned}
\frac{-1}{\frac{1}{m+1} - \frac{1}{m}} &< (m+\delta)^2 \\
\frac{-1}{\frac{-1}{m\,(m+1)}} &< (m+\delta)^2 \\
m^2 + m &< \delta^2 + 2\,m\,\delta + m^2 \\
0 &< \delta^2 + 2\,m\,\delta - m \qquad (3.12)
\end{aligned}$$

In order to solve this quadratic inequality with respect to δ, we initially solve the corresponding equality

$$0 = \delta^2 + 2\,m\,\delta - m \qquad (3.13)$$

and obtain the solution set for (3.13)

$$L_{eq} = \left\{\delta : -m \pm \sqrt{m^2 + m}\right\} \qquad (3.14)$$

We can now factorize (3.12) and it follows

$$0 < \left(\delta + m - \sqrt{m^2 + m}\right)\left(\delta + m + \sqrt{m^2 + m}\right) \tag{3.15}$$

Using case differentiation we obtain the preliminary solution set for (3.12)

$$L' = \left\{\delta : (\delta > \sqrt{m^2 + m} - m) \vee (\delta < -\sqrt{m^2 + m} - m)\right\} \tag{3.16}$$

The second case ($\delta < -\sqrt{m^2 + m} - m$), however, is not a feasible solution in the domain because it violates the requirement $\delta > 0$. The first case is a feasible solution in the domain and it is

$$\sqrt{2} - 1 \leq \delta < \frac{1}{2} \tag{3.17}$$

because

$$\begin{aligned}
&\lim_{m\to\infty}\left(\sqrt{m^2 + m} - m\right)\\
&= \lim_{m\to\infty} \frac{(\sqrt{m^2 + m} - m)(\sqrt{m^2 + m} + m)}{\sqrt{m^2 + m} + m}\\
&= \lim_{m\to\infty} \frac{m}{\sqrt{m^2 + m} + m}\\
&= \lim_{m\to\infty} \frac{\frac{m}{m}}{\sqrt{\frac{m^2}{m^2} + \frac{m}{m^2}} + \frac{m}{m}}\\
&= \lim_{m\to\infty} \frac{1}{\sqrt{1 + \frac{1}{m}} + 1}\\
&= \frac{1}{2}
\end{aligned}$$

Hence, the final solution set of (3.12) in the domain is

$$L = \left\{\delta : \delta > \sqrt{m^2 + m} - m\right\} \tag{3.18}$$

This means, assuming the d^2 attenuation model ($\gamma = 2$), the power needed to relay a stream using K_{floor} hops will be greater than the power needed when using K_{ceil} hops if $\delta > \sqrt{m^2 + m} - m$. Hence, we will choose $K_{opt} = K_{ceil}$ in this case, and $K_{opt} = K_{floor}$ if $\delta < \sqrt{m^2 + m} - m$. We can define a criterion for the d^2 attenuation model:

$$K_{opt} = \begin{cases} K_{floor} & \text{, if } \delta \leq \delta' \\ K_{ceil} & \text{, if } \delta > \delta' \end{cases} \tag{3.19}$$

where

$$\delta' = \sqrt{m^2 + m} - m. \tag{3.20}$$

3.3.3 Criterion for Quartic Attenuation Model

For the case $\gamma = 4$, (3.11) becomes

$$\begin{aligned}
\frac{-3}{\frac{1}{(m+1)^3} - \frac{1}{m^3}} &< (m+\delta)^4 \\
\frac{-3}{\frac{-3\,m^2-3\,m-1}{m^3(m+1)^3}} &< (m+\delta)^4 \\
\frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1} &< \delta^4 + 4\,m\,\delta^3 + 6\,m^2\,\delta^2 + 4\,m^3\,\delta + m^4 \\
0 &< \delta^4 + 4\,m\,\delta^3 + 6\,m^2\,\delta^2 + 4\,m^3\,\delta + m^4 - \frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1}
\end{aligned} \tag{3.21}$$

In order to solve this quartic inequality with respect to δ, we again solve the corresponding equality

$$0 = \delta^4 + 4\,m\,\delta^3 + 6\,m^2\,\delta^2 + 4\,m^3\,\delta + m^4 - \frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1} \tag{3.22}$$

As suggested in [Hah05], we initially convert (3.22) to a *depressed quartic*, that is, we eliminate the δ^3 term. The substitution we make is $\delta = z - m$. When we multiply it out we get the depressed quartic

$$0 = z^4 - \frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1} \tag{3.23}$$

After obtaining the solutions for (3.23) and reversing the substitution, we get the solution set for (3.22)

$$L_{eq} = \left\{ -m \pm \sqrt[4]{\frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1}},\, -m \pm \sqrt[4]{\frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1}}\, i \right\} \tag{3.24}$$

We can now rewrite the inequality (3.21) as a product of real factors

$$0 < \left(\delta - \sqrt[4]{\frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1}} + m\right)\left(\delta + \sqrt[4]{\frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1}} + m\right) R \tag{3.25}$$

where

$$R = \left(\delta^2 + 2\,\delta\,m + m^2 + \sqrt{\frac{3\,m^3\,(m+1)^3}{3\,m^2 + 3\,m + 1}}\right) \tag{3.26}$$

Note that R is the product of the two complex roots of (3.22) and it applies $R > 0$ in the domain. Using case differentiation we obtain the preliminary solution set for (3.21)

$$L' = \left\{\delta : \left(\delta > \sqrt[4]{\frac{3\,m^3\,(m+1)^3}{3\,m^2 + 3\,m + 1}} - m\right) \vee \left(\delta < -\sqrt[4]{\frac{3\,m^3\,(m+1)^3}{3\,m^2 + 3\,m + 1}} - m\right)\right\} \tag{3.27}$$

The second case is not a feasible solution in the domain because it violates the requirement $\delta > 0$. The first case is a feasible solution and it is

$$\sqrt[4]{\frac{24}{7}} - 1 \leq \delta < \frac{1}{2} \tag{3.28}$$

because (with $Q = \frac{2}{3}m^2 - \frac{1}{3}m + \frac{1}{9} - \frac{1}{27m^2+27m+9}$)

$$
\begin{aligned}
&\lim_{m\to\infty}\left(\sqrt[4]{\frac{3m^3(m+1)^3}{3m^2+3m+1}} - m\right) \\
&= \lim_{m\to\infty} \frac{\left(\sqrt[4]{\frac{3m^3(m+1)^3}{3m^2+3m+1}} - m\right)\left(\sqrt[4]{\frac{3m^3(m+1)^3}{3m^2+3m+1}} + m\right)}{\sqrt[4]{\frac{3m^3(m+1)^3}{3m^2+3m+1}} + m} \\
&= \lim_{m\to\infty} \frac{\sqrt{\frac{3m^3(m+1)^3}{3m^2+3m+1}} - m^2}{\sqrt[4]{\frac{3m^3(m+1)^3}{3m^2+3m+1}} + m} \\
&= \lim_{m\to\infty} \frac{\left(\sqrt{\frac{3m^3(m+1)^3}{3m^2+3m+1}} - m^2\right)\left(\sqrt{\frac{3m^3(m+1)^3}{3m^2+3m+1}} + m^2\right)}{\left(\sqrt[4]{\frac{3m^3(m+1)^3}{3m^2+3m+1}} + m\right)\left(\sqrt{\frac{3m^3(m+1)^3}{3m^2+3m+1}} + m^2\right)} \\
&= \lim_{m\to\infty} \frac{\frac{3m^3(m+1)^3}{3m^2+3m+1} - m^4}{\left(\frac{3m^3(m+1)^3}{3m^2+3m+1}\right)^{\frac{3}{4}} + m^2\left(\frac{3m^3(m+1)^3}{3m^2+3m+1}\right)^{\frac{1}{4}} + m\left(\frac{3m^3(m+1)^3}{3m^2+3m+1}\right)^{\frac{1}{2}} + m^3} \\
&= \lim_{m\to\infty} \frac{2m^3 + Q}{(m^4+2m^3+Q)^{\frac{3}{4}} + m^2(m^4+2m^3+Q)^{\frac{1}{4}} + m\,(m^4+2m^3+Q)^{\frac{1}{2}} + m^3} \\
&= \lim_{m\to\infty} \frac{2 + \frac{Q}{m^3}}{\left(1+\frac{2}{m}+\frac{Q}{m^4}\right)^{\frac{3}{4}} + \left(1+\frac{2}{m}+\frac{Q}{m^4}\right)^{\frac{1}{4}} + \left(1+\frac{2}{m}+\frac{Q}{m^4}\right)^{\frac{1}{2}} + 1} \\
&= \frac{2}{1+1+1+1} \\
&= \frac{1}{2}
\end{aligned}
\tag{3.29}
$$

Hence, the final solution set of (3.21) in the domain is

$$
L = \left\{\delta : \delta > \sqrt[4]{\frac{3m^3(m+1)^3}{3m^2+3m+1}} - m\right\} \tag{3.30}
$$

We can define a criterion for the d^4 attenuation model:

$$
K_{opt} = \begin{cases} K_{opt(floor)} & \text{, if } \delta \le \delta' \\ K_{opt(ceil)} & \text{, if } \delta > \delta' \end{cases} \tag{3.31}
$$

with

$$
\delta' = \sqrt[4]{\frac{3m^3(m+1)^3}{3m^2+3m+1}} - m \tag{3.32}
$$

3.3.4 Summary

Given the distance D from a sensor to the base station and the characteristic distance d_{char}, the result of D over d_{char} is the theoretical optimal number of hops to be used when relaying sensor data to the base station. That is, the total energy consumption is minimal. However, practically, the number of hops must be integral. If D is not an integral multiple of d_{char}, we therefore have to choose between the two closest integers, K_{floor} and K_{ceil}, which result from applying the floor and the ceiling function to D/d_{char}, respectively.

For this non-trivial case we derived a criterion that allows us to decide which of the two possibilities is in fact the optimal number of hops, or better said, comes as close as possible to the theoretical optimum. Putting the two criteria from the previous sections together, we can give this criterion as follows,

$$K_{opt} = \begin{cases} K_{floor} & \text{, if } \delta \leq \delta' \\ K_{ceil} & \text{, if } \delta > \delta' \end{cases} \tag{3.33}$$

where

$$\delta' = \begin{cases} \sqrt{m^2 + m} - m & \text{, if } \gamma = 2 \\ \sqrt[4]{\frac{3\,m^3\,(m+1)^3}{3\,m^2+3\,m+1}} - m & \text{, if } \gamma = 4 \end{cases} \tag{3.34}$$

The numbers m and δ are the integral and floating point part of D/d_{char}, respectively. Depending on the assumed path loss exponent γ, we can compute a threshold (δ') using m. Based on the comparison of δ with δ' we can then decide whether $K_{opt(floor)} = m$ or $K_{opt(ceil)} = m + 1$ hops result in the most energy-efficient relaying process.

The advantage of (3.33) over (3.7) is that a sensor node must not know all radio parameters and the path loss exponent to decide whether K_{floor} or K_{ceil} leads to minimum total power consumption. For a specific propagation environment, only the distance D to the base station and the characteristic distance d_{char} are required[1].

We now know the exact optimal number of hops (and also the optimal number of relays) for path loss exponent 2 and 4. We use these findings to compare MERR with the optimal case in Section 5.

[1] The choice of the path loss exponent γ reflects the propagation environment in our model. If sensors do not move and the environment is stable, both the characteristic distance as well as the distance to the base station are constants. These two parameters can be programmed into the sensors after deployment.

Chapter 4

Minimum Energy Relay Routing

In this section, we present the basic concept of the MERR protocol and discuss a possible refinement.

4.1 Approximating Optimal Routing Paths

As discussed in Section 3.3, routing data from a sensor to the base station is then most energy-efficient, if a certain optimal number of nodes are used as relays and the distances between these relays are all equal. In a real linear sensor network, however, it is usually not possible to find such an optimal route. We can only try to approximate the optimal case.

This is the basic idea of our proposed protocol. *Given an arbitrary linear sensor network, MERR finds a route from each sensor to the base station that approximates the optimal routing path.* Finding a route is a synonym for selecting appropriate relays between a sensor and the base station.

In MERR, this selection is made in a distributed manner. *Each sensor seeks independently for that downstream node within its maximum transmission range whose distance is closest to the characteristic distance.* Once all sensors have decided on their respective *next-hop node*, they adjust their transmission power to the lowest possible level such that the radio signal can still be received by this node without any errors. In operation, a sensor transmits always to its preassigned next-hop node, regardless of whether it is data received from other upstream nodes or data obtained by its own sensor readings.

In order to select the best fitting next-hop node, a sensor must know

- the characteristic distance, and
- all distances to downstream nodes within the sensor's maximum transmission range.

In our network model of homogeneous nodes and for a given propagation environment, the characteristic distance is a predefined constant and can be programmed into the sensors during a setup phase. As for the distances to downstream nodes, several approaches are possible (see Section 2.1).

How MERR approximates the optimal routing path is shown as an iterative sequence in Figure 4.1. Note that sensors do not necessarily transmit to their direct downstream neighbor; some nodes can be left out between successive relays. The figure for the distance to the next-hop node is the characteristic distance.

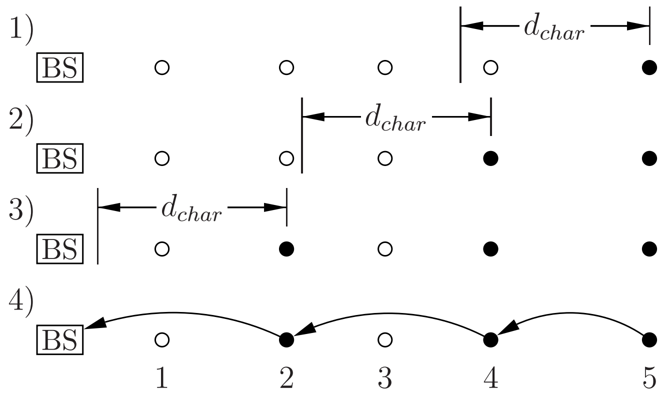


Figure 4.1: Operation of MERR. In steps 1) to 3), the relays 4, 2, and BS are selected. The resulting path $5 \rightarrow 4 \rightarrow 2 \rightarrow$ BS approximates the optimal case and is used in step 4) to route data from sensor 5 to the base station.

It is clear that the given sensor distribution has a significant impact on the performance of our proposed protocol. If the distances between adjacent sensors are all greater than the characteristic distance, each sensor will select its direct downstream neighbor as the next-hop node. In this particular case, MERR is equivalent to MTE routing. Similarly, MERR will establish an optimal route if there exists a sequence of nodes between a sensor and the base station that are spaced in intervals of the characteristic distance.

Proposition 1. *In terms of total power consumption, optimal routing is the lower bound and MTE routing is the upper bound of MERR.*

Proposition 2. *MERR achieves power savings compared to MTE routing if the distances between adjacent sensors are smaller than the characteristic distance.*

Proposition 3. *MERR consumes minimum total power if there exists an optimal routing path.*

We confirm these observations in Section 5 by both theoretical analysis and simulation results.

4.2 Equal Hop Distances

In MERR, sensors use the characteristic distance to select the best fitting next-hop node. A sensor assumes, in a sense, that its distance D to the base station is an integral multiple of d_{char} and thus $K_{opt} = D/d_{char}$. This simplification leads in general to a hangover distance left for the final hop to the base station. That is, the last hop distance is not approximately made equal (to d_{char}).

Consider a sensor that is aware of its distance D to the base station. The optimal number of hops can be determined via

$$K_{opt} = g\left(\frac{D}{d_{char}}\right), \tag{4.1}$$

where $g(x)$ represents either the floor or the ceiling function. The criterion (3.33) can be used to implement $g(x)$. With (4.1), the desired distance to the next-hop node (d_{next}) is given by

$$d_{next} = \frac{D}{K_{opt}}. \tag{4.2}$$

In this way, a sensor factors in its distance to the base station when selecting the next-hop node, and all hop distances are approximately made equal (to d_{next}).

As our simulation results show, this strategy saves power compared to the original MERR protocol. However, since the implementation requires the sensors to be aware of their distances to the base station and the benefit is marginal for linear networks with many sensors ($\approx 1\%$ for $n = 100$), we only consider the original version of MERR in the evaluation.

Chapter 5

Evaluation

We first introduce the stochastic model used to describe the distribution of sensors. Based on this model, we give the expected power consumption for each protocol in the subsequent section, followed by a joint discussion of both theoretical and simulation results.

5.1 Stochastic Model for Sensor Distribution

In our evaluation, we want to be independent of any particular node placement. Sensors should be haphazardly distributed on a line with no obvious regularity or trends in density. The presence of sensor nodes in one section should not influence those in another. *Poisson processes* are models to describe this sort of highly random behavior [Kin93].

5.1.1 Poisson Node Model

For our purpose, we use a *one-dimensional homogeneous Poisson process* to model the distribution of sensors. The characterizing parameter of a homogeneous Poisson process is its constant rate λ.

Definition 1 (**Poisson Node Model**). *The points of a Poisson process with constant rate λ,*

$$0 < X_1 < X_2 < X_3 < \dots, \tag{5.1}$$

represent a random sequence of sensors distributed on a straight line, where X_i is the distance of sensor i to the base station which is located at the origin. We refer to this model as the Poisson Node Model.

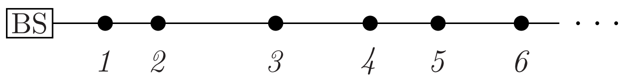


Figure 5.1: In the Poisson Node Model, the points of a one-dimensional homogeneous Poisson process are interpreted as a random sequence of sensors that are placed on a straight line with the base station located at the origin.

5.1.2 Important Characteristics

A Poisson model is usually the simplest and, in a sense, the most random way in which to describe any particular phenomenon [Kin93]. The homogeneous Poisson process in the first dimension is especially convenient, because some relations can be described by well-known probability distributions. We present those relations that are essential for our analysis in the following. The interested reader may be referred to [Kin93] for a comprehensive discussion of Poisson processes. An introduction to probability theory and stochastic processes can be found, for example, in [Och90] and [Ber93].

The distances between sensors play an important role in our analysis, because they determine the required transmission power. The following theorem makes a statement about how we can describe the distances between adjoining sensors.

Theorem 1 (Interval Theorem). *According to the Poisson Node Model, the random variables*

$$Y_1 = X_1, \qquad Y_i = X_i - X_{i-1}, \qquad i \geq 2, \tag{5.2}$$

represent the distances between adjoining sensors. These distances are stochastically independent, and each has an exponential distribution with associated parameter λ.

Definition 2 (Exponential Distribution). *A continuous random variable X has the* exponential distribution $Exp(\lambda)$ *if its probability density function is given by*

$$f_X(x) = \lambda\, e^{-\lambda x} \qquad x \geq 0,\ \lambda > 0. \tag{5.3}$$

Mean and variance of $Exp(\lambda)$ are as follows:

$$\mathbb{E}[X] = \frac{1}{\lambda}, \tag{5.4}$$

$$\mathrm{var}[X] = \frac{1}{\lambda^2}. \tag{5.5}$$

We can thus describe each distance between adjoining sensors with a random variable that has an exponential distribution $Exp(\lambda)$. Here, parameter λ corresponds to the rate of the underlying Poisson process Π. The same holds for the gamma distribution that comes into play when we look at the distance between a sensor and the base station.

Theorem 2 (**Distance Theorem**). *In the Poisson Node Model, the random variables X_i, representing the distance of sensor i to the base station, are stochastically independent, and each has gamma distribution with associated parameters λ and i for all $i \geq 1$.*

Definition 3 (**Gamma Distribution**). *A continuous random variable X has the* gamma distribution *$Gamma(m, \lambda)$ if its probability density function is given by*

$$f_X(x) = \frac{\lambda^m \, x^{m-1}}{\Gamma(m)} e^{-\lambda x}, \qquad x \geq 0, \, \lambda > 0, \tag{5.6}$$

where $\Gamma(m)$ is the gamma function given by

$$\Gamma(m) = \int_0^\infty x^{m-1} \, e^{-x} \, dx. \tag{5.7}$$

Mean and variance of $Gamma(m, \lambda)$ are as follows:

$$\mathbb{E}[X] = \frac{m}{\lambda}, \tag{5.8}$$

$$\mathrm{var}[X] = \frac{m}{\lambda^2}. \tag{5.9}$$

We can thus describe the distance of sensor i to the base station with a random variable that has gamma distribution $Gamma(i, \lambda)$ for all $i \geq 1$. In fact, due to the independent structure of the Poisson process, the distance between any two sensors i and j with $i < j$ has a gamma distribution $Gamma((j - i), \lambda)$.

Now we have all stochastic tools that we need to analyze direct transmission, MTE routing, and MERR based on the Poisson Node Model.

5.2 Theoretical Analysis

We derive the expected power consumption of each protocol for the transmission of one bit from sensor n to the base station.

Theorem 3 (**Expected Power Direct Transmission**). *For the Poisson Node Model, the expectation of power needed to transmit one bit from node n to the base* *station using direct transmission is given by*

$$\mathbb{E}[P_{DT}(X_n)] \;=\; \alpha_{tx} + \frac{\Gamma(n+\gamma)\,\epsilon}{\Gamma(n)\,\lambda^\gamma}, \tag{5.10}$$

where X_n is the distance of node n to the base station.

Proof. The expectation of (transmission) power needed at node n can be written as follows:

$$\begin{aligned}\mathbb{E}[P_{DT}(X_n)] &= \mathbb{E}[\alpha_{tx} + \epsilon\,(X_n)^\gamma] \\ &= \alpha_{tx} + \epsilon\,\mathbb{E}[(X_n)^\gamma].\end{aligned}$$

From Theorem 2 we know that X_n has gamma distribution $Gamma(n, \lambda)$. Thus we get

$$\begin{aligned}\mathbb{E}[P_{DT}(X_n)] &= \alpha_{tx} + \epsilon \int_0^\infty x^\gamma\, f_X(x)\mathrm{d}x \\ &= \alpha_{tx} + \epsilon\,\frac{\lambda^n}{\Gamma(n)} \int_0^\infty x^{n+\gamma-1}\, e^{-\lambda\, x}\mathrm{d}x \\ &= \alpha_{tx} + \epsilon\,\frac{\lambda^n}{\Gamma(n)}\frac{\Gamma(n+\gamma)}{\lambda^{n+\gamma}} \\ &= \alpha_{tx} + \frac{\Gamma(n+\gamma)\,\epsilon}{\Gamma(n)\,\lambda^\gamma},\end{aligned}$$

which proofs the theorem. ∎

Theorem 4 (**Expected Power MTE Routing**). *For the Poisson Node Model, the expectation of power needed to transmit one bit from node n to the base station using MTE routing is given by*

$$\mathbb{E}[P_{MTE}(X_n)] \;=\; -\alpha_{rx} + n\left(\alpha + \frac{\Gamma(\gamma+1)\,\epsilon}{\lambda^\gamma}\right), \tag{5.11}$$

where X_n is the distance of node n to the base station.

Proof. We can express the expectation of power for MTE routing as follows:

$$\begin{aligned}\mathbb{E}[P_{MTE}(X_n)] &= \mathbb{E}[-\alpha_{rx} + \textstyle\sum_{i=1}^n P_{relay}(Y_i)] \\ &= -\alpha_{rx} + \mathbb{E}[\textstyle\sum_{i=1}^n P_{relay}(Y_i)] \\ &= -\alpha_{rx} + n\,\mathbb{E}[P_{relay}(Y)]. \qquad (5.12)\end{aligned}$$

In the last step we take advantage of the fact that the random variables Y_i, and hence also the random variables $P_{relay}(Y_i)$, are stochastically independent and have the same distribution for all $i \geq 1$. Then we have

$$\begin{aligned} \mathbb{E}[P_{relay}(Y)] &= \mathbb{E}[\alpha + \epsilon\, Y^{\gamma}] \\ &= \alpha + \epsilon\, \mathbb{E}[Y^{\gamma}]. \end{aligned} \tag{5.13}$$

and with Theorem 1, which says that random variable Y has exponential distribution $Exp(\lambda)$, we get

$$\begin{aligned} \mathbb{E}[P_{relay}(Y)] &= \alpha + \epsilon \int_0^{\infty} y^{\gamma}\, f_Y(y) \mathrm{d}y \\ &= \alpha + \epsilon\, \lambda \int_0^{\infty} y^{\gamma}\, e^{-\lambda\, y} \mathrm{d}y \\ &= \alpha + \epsilon\, \lambda\, \frac{\Gamma(\gamma + 1)}{\lambda^{\gamma+1}} \\ &= \alpha + \frac{\Gamma(\gamma + 1)\, \epsilon}{\lambda^{\gamma}}. \end{aligned} \tag{5.14}$$

Inserting (5.14) in (5.12) yields to the proposition. ■

Theorem 5 (Expected Power MERR Routing). *For the Poisson Node Model, the expectation of power needed to transmit one bit from node n to the base station using MERR is given by*

$$\mathbb{E}[P_{MERR}(X_n)] = -\alpha_{rx} + \frac{n}{\lambda\, d_{char}}\, \mathbb{E}[P_{relay}(Y)], \tag{5.15}$$

where X_n is the distance of node n to the base station, Y the distance between successive relays, and

$$\mathbb{E}[P_{relay}(Y)] = \alpha + \frac{\epsilon\, \omega^{n}}{\lambda^{\gamma}\, \Gamma(n)} \int_0^{\infty} \frac{\Gamma(s+\gamma)}{\Gamma(s)}\, s^{-(n+1)} e^{\omega s^{-1}} \mathrm{d}s \tag{5.16}$$

where s is the number of intermediate hops between successive relays and $\omega = \lambda\, n\, d_{char}$.

Proof. Assuming that the distances between successive relays are on average equal to the characteristic distance, random variable

$$S = \frac{n\, d_{char}}{X_n} \tag{5.17}$$

describes the number of intermediate hops between successive relays. Random variable S has inverse gamma distribution $InvGamma(n, \omega)$ with probability density given by

$$f_S(s) \;=\; \frac{\omega^n}{\Gamma(n)}\, s^{-(n+1)}\, e^{-\omega s^{-1}}, \tag{5.18}$$

where $\omega = \lambda\, n\, d_{char}$. Since the distance between any two sensors in the Poisson Node Model has a gamma distribution, we can describe the distances Z between successive relays by gamma distribution $Gamma(S, \lambda)$. Furthermore, the transmission between successive relays is basically a direct transmission, whereas, contrary to (5.10), also receive energy is expended (except for the last transmission to the base station). Thus, we deduce the conditional expectation of power needed at each relay,

$$\mathbb{E}[P_{relay}(Z)|S = s] \;=\; \alpha + \frac{\Gamma(s+\gamma)\,\epsilon}{\Gamma(s)\,\lambda^\gamma}, \tag{5.19}$$

and hence with

$$\mathbb{E}[P_{relay}(Z)] \;=\; \int_0^\infty \mathbb{E}[P_{relay}(Z)|S = s] \cdot f_S(s)\,\mathrm{d}s \tag{5.20}$$

we get (5.16). Now, let random variable K be the number of hops from node n to the base station using all relays. The expectation of K is

$$\mathbb{E}[K] = \frac{n}{\lambda\, d_{char}}, \tag{5.21}$$

and with

$$\begin{aligned}\mathbb{E}[P_{MERR}(X_n)] \;&=\; \mathbb{E}[-\alpha_{rx} + \textstyle\sum_{i=1}^{K} P_{relay}(Z)] \\ &=\; -\alpha_{rx} + \mathbb{E}[K]\,\mathbb{E}[P_{relay}(Z)]\end{aligned} \tag{5.22}$$

we get (5.15) which concludes the proof. ■

To the best of our knowledge, the integral in (5.16) is not completely solvable for $\gamma \in \mathbb{R}$. Typically, γ ranges between 1.6 and 6 depending on the propagation environment [Rap96], but in the discussion below we consider only two common choices where $\gamma = 2$ or $\gamma = 4$. For these integer values we get exact solutions. These are

$$\mathbb{E}[P_{relay}(Y)] \;=\; \alpha + \epsilon\left[C + \frac{1}{\lambda}\, D\right] \tag{5.23}$$

for $\gamma = 2$ and

$$\mathbb{E}[P_{relay}(Y)] \;=\; \alpha + \epsilon\left[A + \frac{6}{\lambda}\, B + \frac{11}{\lambda^2}\, C + \frac{6}{\lambda^3}\, D\right] \tag{5.24}$$

for $\gamma = 4$ where

$$A = \frac{(n\, d_{char})^4}{(n-1)(n-2)(n-3)(n-4)} \tag{5.25}$$

$$B = \frac{(n\, d_{char})^3}{(n-1)(n-2)(n-3)} \tag{5.26}$$

$$C = \frac{(n\, d_{char})^2}{(n-1)(n-2)} \tag{5.27}$$

$$D = \frac{n\, d_{char}}{n-1}. \tag{5.28}$$

In fact, the integral in (5.16) is completely solvable for all $\gamma \in \mathbb{N}$. In that case, the argument of the gamma function is a positive integer and hence the gamma function can be replaced by the factorial. For other values of γ, numerical methods must be employed.

Theorem 6 (**Expected Power Optimal Routing**). *For the Poisson Node Model, the minimum expected power consumption for transmitting one bit from node n to the base station is given by*

$$\mathbb{E}[P_{OPT}(X_n)] = -\alpha_{rx} + K_{opt}\left[\alpha + \epsilon\left(\frac{n}{\lambda\, K_{opt}}\right)^{\gamma}\right], \tag{5.29}$$

where X_n is the distance of node n to the base station and K_{opt} is given by (3.33) with $D = n/\lambda$ for $\gamma = 2$ or $\gamma = 4$.

Proof. The optimal routing path from sensor n consists of $(K_{opt} - 1)$ relays that are spaced in constant intervals (see Section 3.3). Assuming that the optimal number of hops K_{opt} is given by criterion (3.33), the constant distance between successive relays is

$$Y = \frac{D}{K_{opt}} = \frac{n}{\lambda\, K_{opt}}. \tag{5.30}$$

Thus, we can express the expectation of power for optimal routing as follows:

$$\begin{aligned} \mathbb{E}[P_{OPT}(X_n)] &= \mathbb{E}[-\alpha_{rx} + \textstyle\sum_{i=1}^{K_{opt}} P_{relay}(Y)] \\ &= -\alpha_{rx} + \mathbb{E}[\textstyle\sum_{i=1}^{K_{opt}} P_{relay}(Y)] \\ &= -\alpha_{rx} + K_{opt}\, \mathbb{E}[P_{relay}(Y)] \end{aligned} \tag{5.31}$$

Since the distances between successive relays Y are all the same, also the power rates $P_{relay}(Y)$ dissipated at each relay are the same. Consequently, we have

$$\begin{aligned} \mathbb{E}[P_{relay}(Y)] &= \mathbb{E}[\alpha + \epsilon\, Y^\gamma] \\ &= \alpha + \epsilon\, Y^\gamma \\ &= \alpha + \epsilon \left(\frac{n}{\lambda\, K_{opt}}\right)^\gamma . \end{aligned} \tag{5.32}$$

Inserting (5.32) in (5.31) leads to the proposition. ∎

5.3 Discussion of Results

In addition to a theoretical analysis, we simulated direct transmission, MTE routing, and the MERR protocol to determine the power a specific protocol needs to deliver one bit from sensor n to the base station. The radio characteristics (adopted from [Hei00]) and network parameters are summarized in Table 5.1. The choice of $n = 100$ sensors is reasonable for installations along pipelines or long bridges. For these radio parameters, we get the characteristic distances via (3.3) as listed in Table 5.2. Since the results of both theoretical analysis and simulation yet emphasize the same facts, we discuss them together in this section. Note that, because of high variations in the sensor distributions generated for the experiments, we averaged the values of 10 sample networks (generated with the same Poisson rate) to even the simulation results.

Consider the two plots in Figure 5.2 that were generated using the expressions from Section 5.2. They give a performace overview of all protocols for path loss exponent 2. The curves in Figure 5.2(a) show the dependency of power consumption on Poisson rate λ for the same number of nodes, while Figure 5.2(b) shows the dependency on the number of nodes n for a certain Poisson rate.

It can be seen that direct transmission is not energy-efficient if the distance to the base station is long, since the power consumption scales both with λ to the power of γ and with n to the power of γ. All other protocols have a linear dependency on n. Next, we note that MERR is bounded by MTE routing (upper bound) and optimal routing (lower bound). As shown in Figure 5.2(a), MERR never consumes more power than MTE routing but approaches the theoretical optimum. This confirms Proposition 1 from Section 4. The point at which MERR and MTE routing have

Table 5.1: Radio characteristics and network parameters

Description	Parameter	Value
Receiver electronics energy	α_{rx}	50 nJ/bit
Transmitter electronics energy	α_{tx}	50 nJ/bit
Combined electronics energy	α	100 nJ/bit
Path loss exponent	γ	2 or 4
Radio amplifier energy	ϵ	10 pJ/bit/m^2 ($\gamma = 2$) 0.0013 pJ/bit/m^4 ($\gamma = 4$)
Number of nodes	n	100
Poisson rate	λ	$[0.001, 1]$
Relay rate	r	1 bit/s

Table 5.2: Characteristic distance and Poisson rate

Measure	$\gamma = 2$	$\gamma = 4$
Characteristic distance [m]	100	71
Corresponding Poisson rate	0.010	0.014

approximately equal power consumption is indicated by a vertical line[1]. Here, the Poisson rate corresponds to the characteristic distance.

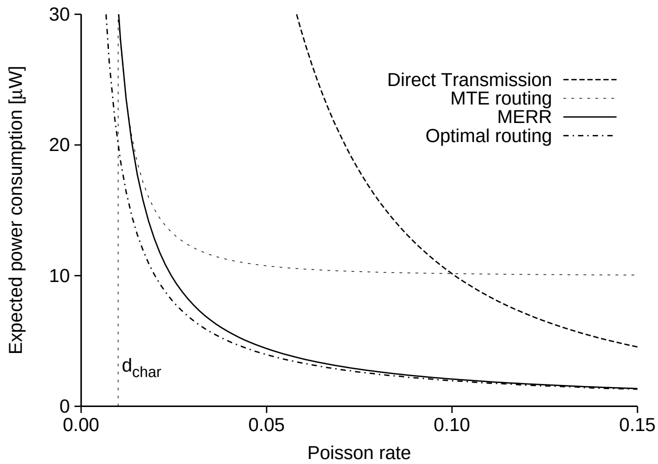
Let us now focus on Figure 5.3 to clarify the relation between MTE routing, MERR, and the characteristic distance. The two curves for path loss exponent $\gamma = 2$ and $\gamma = 4$, respectively, were plotted according to

$$[\text{Power savings}] \;=\; 100 \cdot \left(1 - \frac{P_{MERR}}{P_{MTE}}\right). \tag{5.33}$$

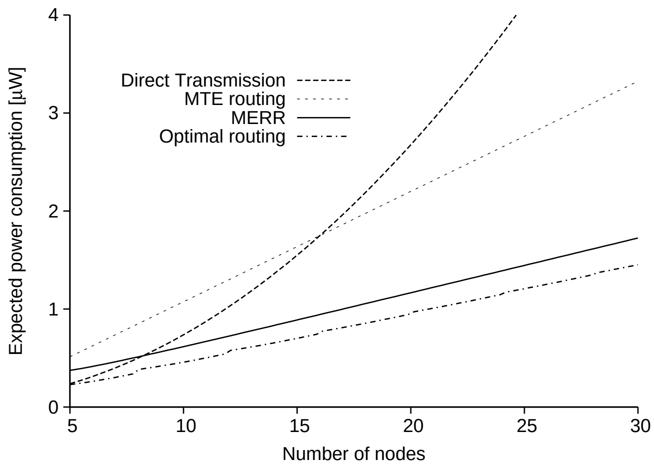
Thus, the graphs show how much power MERR saves relative to MTE routing depending on the Poisson rate. Figure 5.3(b) is a magnification of the rectangular section in Figure 5.3(a).

We can see that the benefit of MERR increases as the Poisson rate λ increases, or equivalently, the mean distance between adjacent sensors $(1/\lambda)$ decreases. Even if the mean distance is slightly smaller than the characteristic distance, MERR is more power-efficient than MTE routing. This behavior results from the high variations in the distances between adjacent sensors in the Poisson Node Model. Although the mean distance is equal to the characteristic distance, some sensor are actually closer to each other. Thus, not all nodes are used as relays in MERR which leads to the

[1] In our simulations, the MERR protocol consumed lightly less power than predicted by the theoretical analysis. Thus, the graph in Figure 5.2(a) does not exactly match the graphs in Figure 5.3.



(a) Expected power consumption depending on Poisson rate for constant number of sensors ($n = 100$).



(b) Expected power consumption depending on number of nodes for constant Poisson rate ($\lambda = 0.04$).

Figure 5.2: Expected power consumption of direct transmission, MTE routing, optimal routing and MERR for path loss exponent 2.

observed savings. The power savings of MERR compared to MTE routing are listed in Table 5.3 for some mean distances. Note that 100 m and 71 m correspond to the characteristic distance for $\gamma = 2$ and $\gamma = 4$, respectively. For a conceivable mean distance of 10 m, MERR consumes about 80% less power. This confirms Proposition 2.

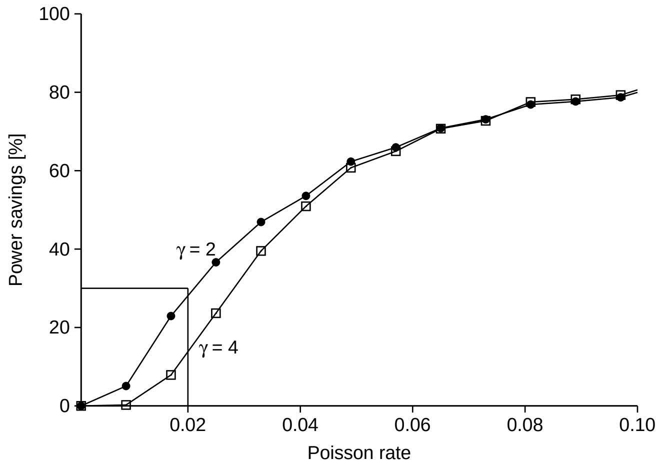
The convergence of MERR against the theoretical optimum is shown in Figure 5.4. The curves were generated according to

$$[\text{Excess consumption}] = 100 \cdot \left(\frac{P_{MERR}}{P_{OPT}} - 1 \right), \tag{5.34}$$

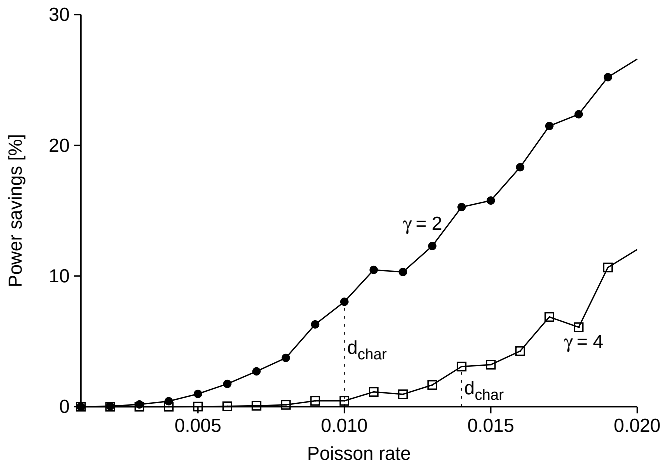
Table 5.3: Power savings of MERR compared to MTE routing depending on the mean distance between adjacent sensors

Mean Distance [m]	$\gamma = 2$ [%]	$\gamma = 4$ [%]
100	7	0
71	15	3
50	27	13
30	47	37
20	63	61
10	81	82

and therefore indicate how much MERR differs from the optimal case (x-axis) in terms of power consumption. Both curves fall off rapidly and differ less than 10% from the optimum if $\lambda > 0.02$ ($\equiv$ 50 m) and $\lambda > 0.04$ ($\equiv$ 24 m) for $\gamma = 2$ and $\gamma = 4$, respectively. MERR finds routes close to the optimal routing path because the probability that a node can find a next-hop node exactly d_{char} onward increases with greater values of λ. This trend confirms Proposition 3.



(a) Power savings (Overview).



(b) Power savings around the characteristic distance.

Figure 5.3: Power savings of MERR relative to MTE routing for path loss exponent 2 and 4.

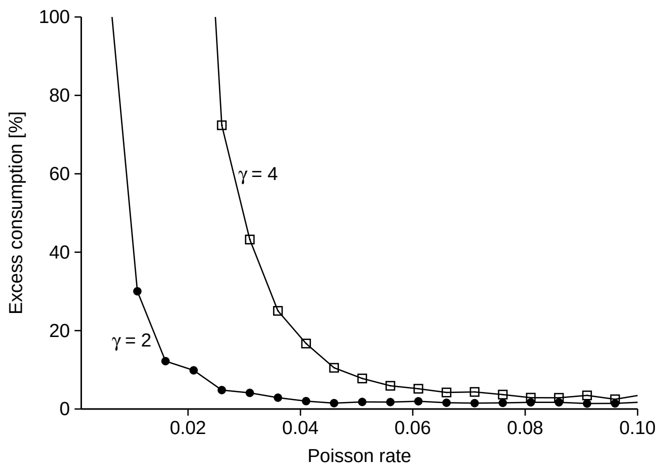


Figure 5.4: Excess consumption of MERR relative to the theoretical optimum. The power consumption of MERR rapidly approaches the minimum as the Poisson rate increases.

Chapter 6

Related Work

To reflect this paper's contribution, we discuss other work on linear sensor and ad hoc networks followed by an assessment of communication protocols for wireless sensor networks that are used in practice.

We exploit some results from [BGC01], where the authors examine upper bounds on the lifetime of data gathering sensor networks. An optimal number of hops is derived that must be used when relaying data from a sensor to the base station most energy-efficiently. This optimal number depends on the total distance to the base station and the characteristic distance, which in turn depends on the propagation environment and the radio parameters. In MERR, the characteristic distance is used to select appropriate relay nodes.

The optimization of transmission range as a system design issue is studied in [COC02]. Assuming a highly dense network with nodes of low mobility, short range, and without power control, the authors argue that the optimum range could be set for all nodes at the system design stage. This optimal one-hop distance is very similar to the characteristic distance. Compared to this work, our study does not make the assumption that the sensors are densely deployed. Also, rather than setting a common transmission range for all nodes in advance, nodes running our proposed protocol set their best possible range individually while in actual operation.

In [GBH+04], a relationship between optimal transmission range and traffic is used to improve the topology management scheme GAF [XHE01]. A heuristic algorithm divides a linear ad hoc wireless network with high node density into non-uniform segments. At any instant, only one node in each segment routes data while the remaining nodes are in sleep state. The authors argue that this strategy counteracts the higher load of nodes close to the base station and thus reduces the total energy consumption. Unlike this work, we do not assume high node density and focus solely on the routing problem in sensor networks

A non-linear programming problem is used in [CCL04] to determine both the locations of sensor nodes and data transmission pattern in order to optimize network lifetime and total power consumption. The authors show that optimal node placement and data transmission pattern leads to a significant benefit over the uniform placement. While this work addresses the placement of sensors on a line, we study the problem of energy-efficient routing for any given linear sensor topology.

Directed diffusion [IGE00] and SPIN [KHB99] are two prominent multi-hop routing protocols for sensor networks used in practice. In directed diffusion, the base station requests data by broadcasting interests that diffuse through the network hop by hop. At the end of this process, gradients are set up from source nodes which possess the desired data back to the base station. When interests fit gradients, paths of information flow are formed from multiple paths, and the most effective paths are reinforced. As for SPIN, all information from each node is disseminated to every node in the network assuming that all nodes shall be able to provide required information immediately. A node advertises new data to its neighbors and transmits the data upon request.

We argue that in both protocols, sensor nodes communicate solely with their one-hop neighbors. With respect to a linear topology, this behavior leads to a scheme that is similar to MTE routing where all intermediate nodes between source node and base station take part in the routing process. Thus, if applied to a linear topology, MERR saves routing energy compared to directed diffusion and SPIN. We believe that MERR can effectively complement directed diffusion. While the base station still requests data via hop by hop interest forwarding, MERR can be used to establish the routes back to the base station.

Chapter 7

Conclusions

In this paper, we introduced MERR (Minimum Energy Relay Routing), a routing protocol specifically designed for linear wireless sensor networks. MERR uses the characteristic distance to establish energy-efficient relay paths to the base station that aim for minimizing the total power consumption.

We discussed optimal routing and the two conventional approaches direct transmission and MTE routing. Based on a stochastic model for the distribution of sensors on a line, we evaluated MERR by comparing it to these three schemes using results from both theoretical analysis and simulation.

After examining MERR in this paper, we arrive at the following conclusions:

- In terms of total power consumption, optimal routing is the lower bound and MTE routing is the upper bound of MERR.
- If the mean distance between adjacent sensors is smaller than the characteristic distance, MERR performs better than MTE routing. Power savings of up to 80% are possible for practicable linear wireless sensor networks.
- MERR's power consumption differs less than 10% from the theoretical minimum if the mean distance between adjacent sensors is smaller than half (50 m) and one third (24 m) of the characteristic distance for path loss exponent 2 and 4, respectively.

In summary, MERR shows significant power savings compared to the conventional approaches and comes close to the theoretical optimum.

In our future work, we would like to study the performance of MERR in interaction with the MAC layer. In particular, we are interested in designing a MAC protocol that complements MERR and includes strategies that enable adaption to node failures and other external influences. We expect from such an integrated approach reliable low-power routing with high data throughput in linear wireless sensor networks.

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Selbstständigkeitserklärung

Hiermit versichere ich, Marco Zimmerling, dass ich die vorliegende Belegarbeit mit dem Thema *Energy-Efficient Routing in Linear Wireless Sensor Networks* selbstständig verfasst habe und keine anderen als die angegebenen Quellen benutzt wurden.

Dresden, 27. April 2007

Marco Zimmerling